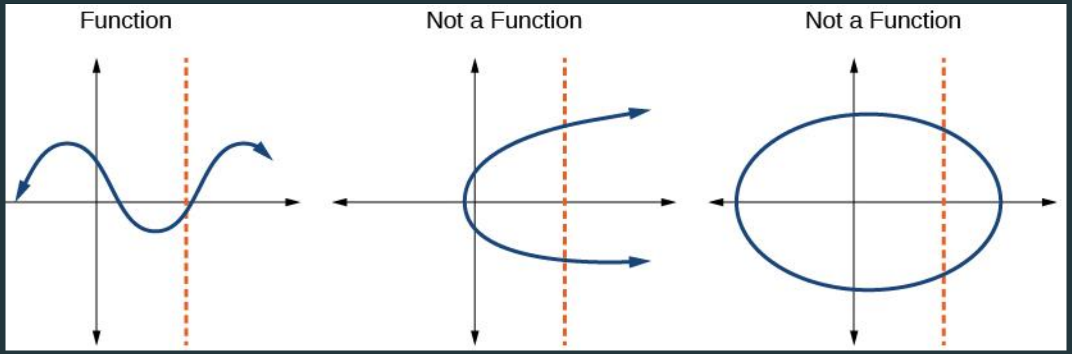
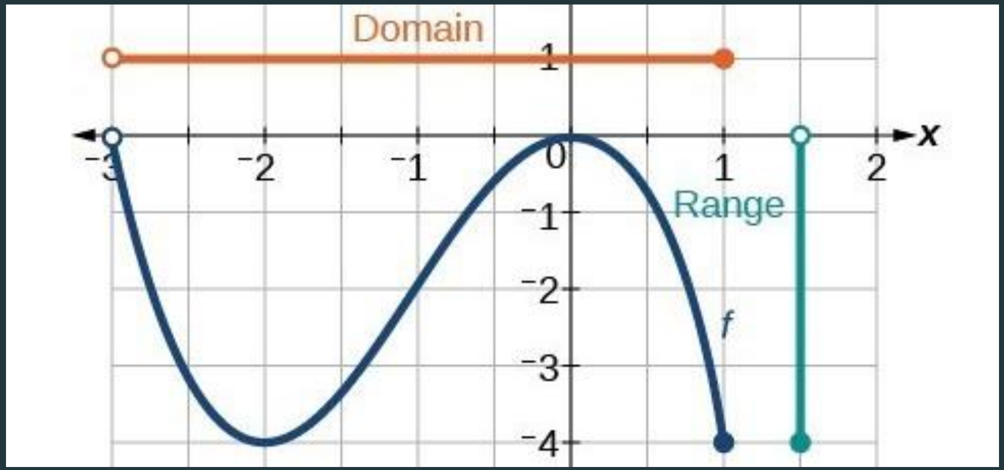
AP Precalc Review: Unit 1

Polynomial and Rational Functions

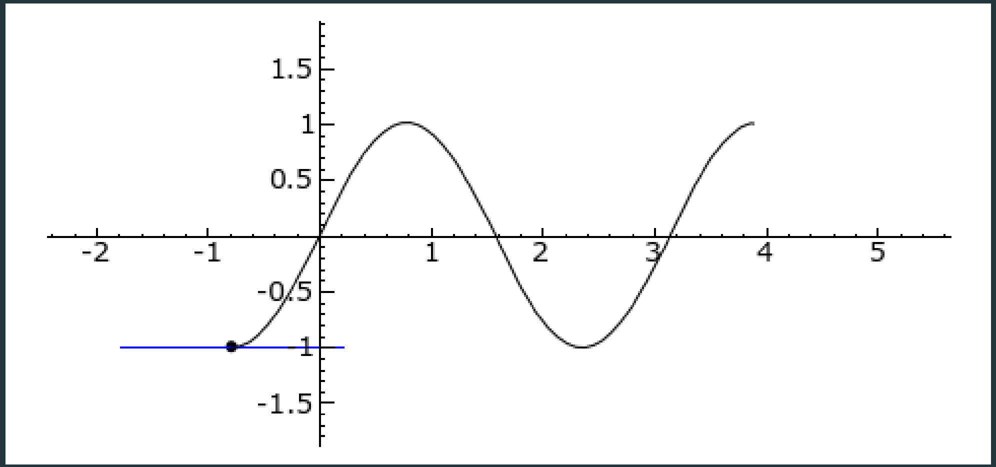
Functions

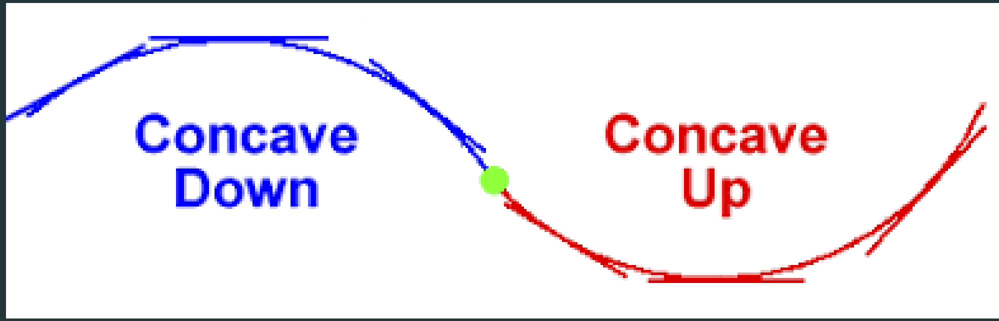




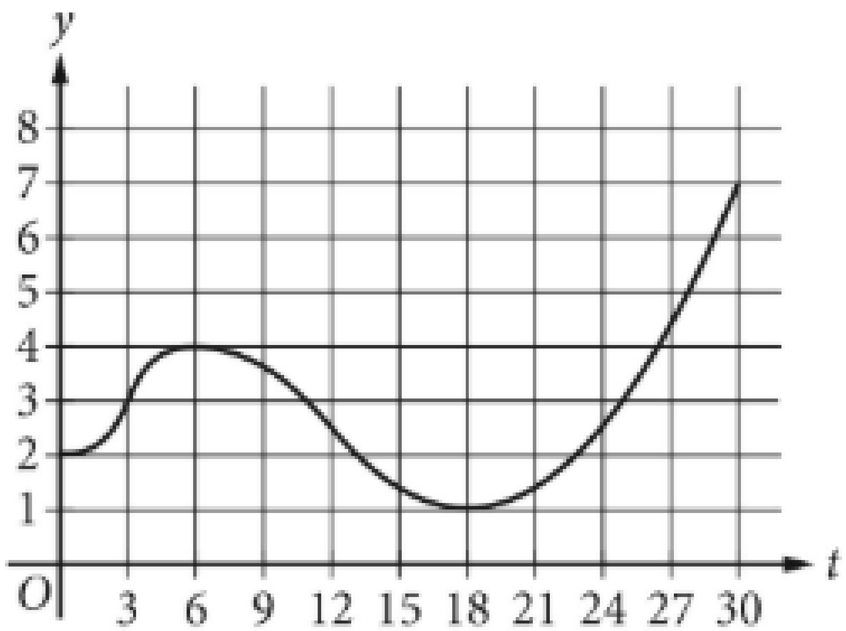
- Function: relationship between input and output
- Input is <u>domain,</u> output is <u>range</u>
- Input values can only have one possible output value, but output values can have multiple input values (vertical line test)

Increasing and decreasing functions





- → Increasing: output values increase as input values increase
- → Decreasing: output values decrease as input values increase
- → Concave up: curves upward; rate of change/slope of tangent is increasing
- → Concave down: curves downward; rate of change/slope of tangent is decreasing
- → Points of inflection: changes in concavity
- → Steeper slope doesn't necessarily mean increasing
- → Zeroes: where graph intersects x-axis (roots, solutions, x-intercepts)



Graph of W

The depth of water, in feet, at a certain place in a lake is modeled by a function W. The graph of $y = W(t)$ is shown for $0 \le t \le 30$, where t is the number of days since the first day of a month. What are all intervals of t on which the depth of water is increasing at a decreasing rate?

(A) $(3, 6)$ only

(B) $(3, 12)$

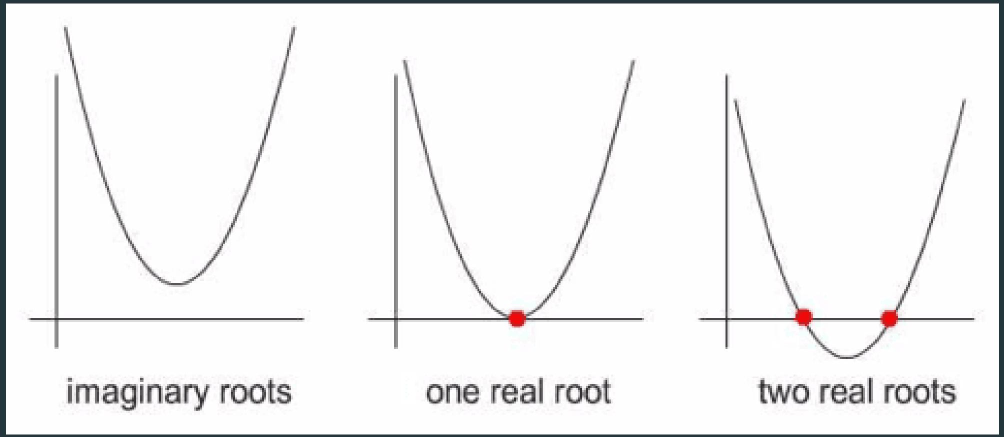
(C) $(0, 3)$ and $(18, 30)$ only

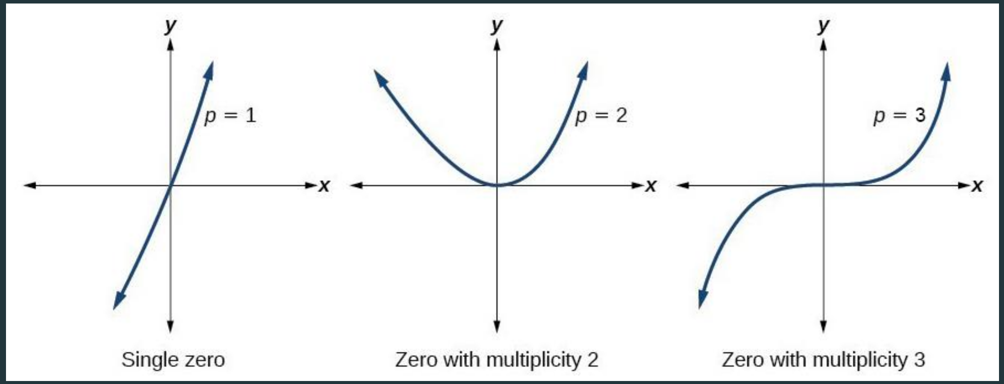
(D) $(0, 6)$ and $(18, 30)$

Change

- → Average rate of change: change over an interval
- → $\frac{y2-y1}{x2-x1}$
- → Linear function: constant rate of change
- → Rate of change of a quadratic has a constant rate of change (second difference)

POLYNOMIALS



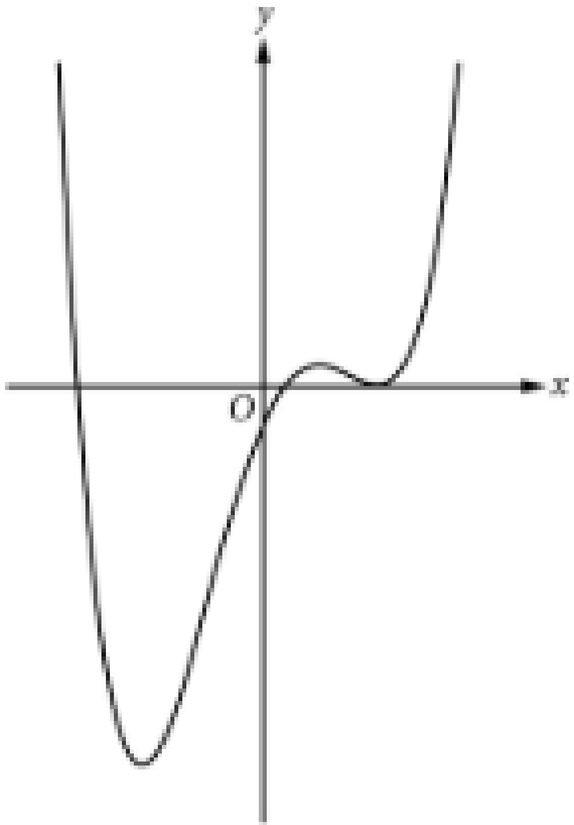


→ Local/relative minima and maxima: points where function changes from increasing to decreasing or decreasing to increasing

→ Global/absolute: highest or lowest points on the graph

→ In polynomials with only real coefficients, every complex zero occurs in a conjugate pair

→ Polynomial long division: polynomial divided by root = 0



Graph of g

The figure shown is the graph of a polynomial function g. Which of the following could be an expression for $g(x)$?

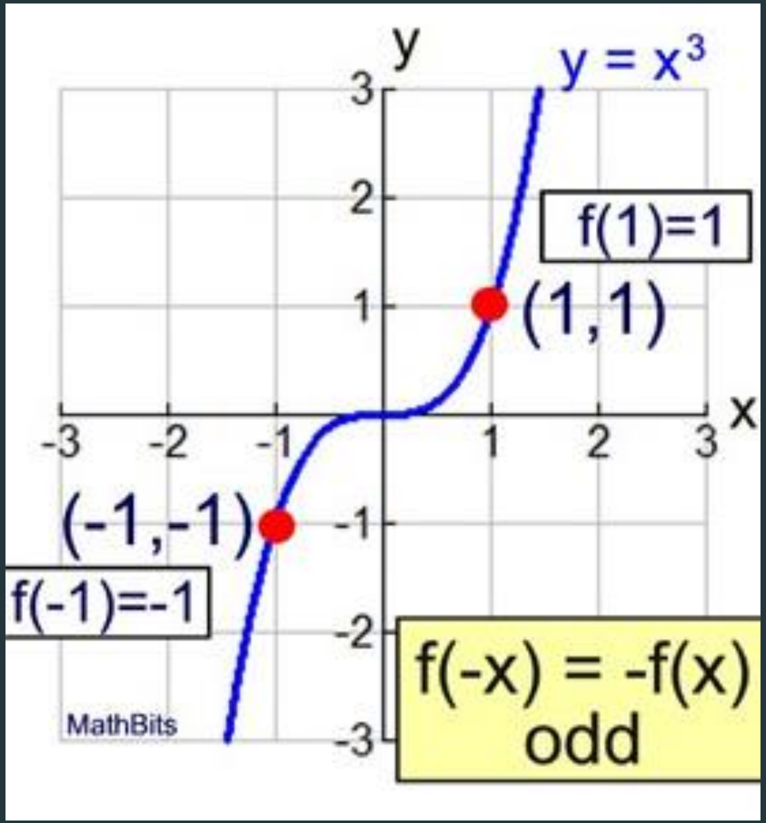
(A) $0.25(x-5)(x-1)(x+8)$

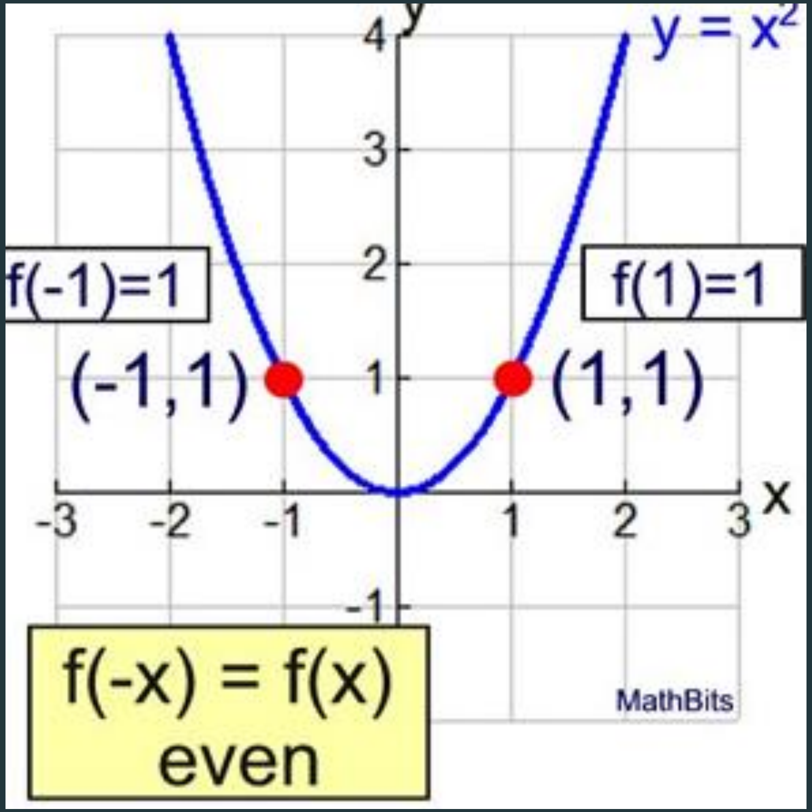
(B) $0.25(x+5)(x+1)(x-8)$

(C) $0.25(x-5)^2(x-1)(x+8)$

(D) $0.25(x+5)^2(x+1)(x-8)$

Even and odd functions





- Even functions: symmetric across y-axis, f(x) = f(-x)
- Odd functions: symmetric at 180 degree rotation about the origin, -f(x) = f(-x)

4. The polynomial function p is an odd function. If $p(3) = -4$ is a relative maximum of p, which of the following statements about $p(-3)$ must be true?

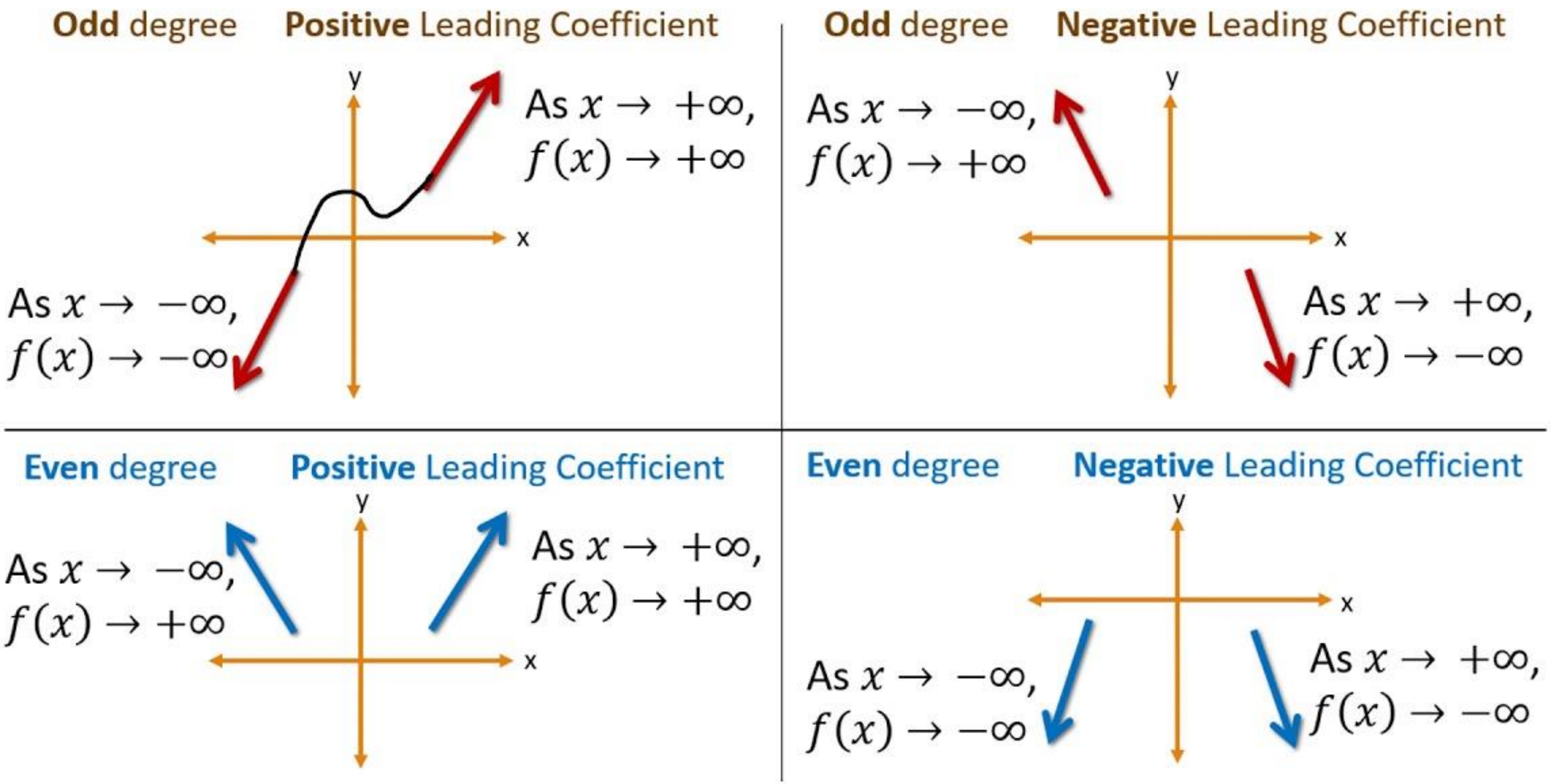
(A) $p(-3) = 4$ is a relative maximum.

(B) $p(-3) = -4$ is a relative maximum.

(C) $p(-3) = 4$ is a relative minimum.

(D) $p(-3) = -4$ is a relative minimum.

End Behavior of a Polynomial Function



The polynomial function p is given by $p(x) = -4x^5 + 3x^2 + 1$. Which of the following statements about the end behavior of p is true?

(A) The sign of the leading term of p is positive, and the degree of the leading term of p is even; therefore, $\lim_{x \to -\infty} p(x) = \infty$ and $\lim_{x \to \infty} p(x) = \infty$.

(B) The sign of the leading term of p is negative, and the degree of the leading term of p is odd; therefore, $\lim_{x \to -\infty} p(x) = \infty$ and $\lim_{x \to \infty} p(x) = -\infty$.

(C) The sign of the leading term of p is positive, and the degree of the leading term of p is odd; therefore, $\lim_{x \to -\infty} p(x) = -\infty$ and $\lim_{x \to \infty} p(x) = \infty$.

(D) The sign of the leading term of p is negative, and the degree of the leading term of p is odd; therefore, $\lim_{x \to -\infty} p(x) = -\infty$ and $\lim_{x \to \infty} p(x) = \infty$.

Rational functions

- End behavior:
  - Leading terms have degree = horizontal asymptote
  - Denominator > numerator = y=0 horizontal asymptote
  - Numerator > denominator = same end behavior as $y = \frac{a}{b}x^{n-d}$
  - Slant asymptote with polynomial long division is n>d by 1
- Holes: factors that cancel out, plug into simplified form to find y-coordinate
- Vertical asymptote: set denominator equal to 0
- Roots: set numerator equal to 0

$$\frac{x^3 + 4x^2 - 12x}{x^2 + 7x + 6}$$

Which of the following functions has a zero at $x = 3$ and has a graph in the xy-plane with a vertical asymptote at $x = 2$ and a hole at $x = 1$?

(A) $h(x) = \dfrac{x^2 - 4x + 3}{x^2 - 3x + 2}$

(B) $j(x) = \dfrac{x^2 - 5x + 6}{x^2 - 3x + 2}$

(C) $k(x) = \dfrac{x - 3}{x^2 - 3x + 2}$

(D) $m(x) = \dfrac{x - 3}{x^2 - 4x + 3}$

Binomial theorem

Exponent	Pascal's Triangle	Binomial Expansion
0	1	$(a+b)^0 = 1$
1	1 1	$(a+b)^1 = 1a + 1b$
2	1 2 1	$(a+b)^2 = 1a^2 + 2ab + 1b^2$
3	1 3 3 1	$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$
4	1 4 6 4 1	$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$
5	1 5 10 10 5 1	$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$
6		

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{n} a^0 b^n$$

$Transformations: g(x) = af\left(b(x-h)\right) + k$

- a: vertical dilation by factor of $|a|$, reflection over x axis if negative
- b: horizontal dilation by factor of $\left|\frac{1}{b}\right|$, reflection over y axis if negative
- k: vertical translation of k units
- h: horizontal translation of –h units

- If VA at x=-2, and HA at y=3 for f(x), find new asymptotes of $g(x)=2f(x+1)-3$

x	-8	-4	-2	-1	0	3
$f(x)$	87	55	5	-4	-7	20

The table gives values for a polynomial function f at selected values of x. Let $g(x) = af(bx) + c$, where a, b, and c are positive constants. In the xy-plane, the graph of g is constructed by applying three transformations to the graph of f in this order: a horizontal dilation by a factor of 2, a vertical dilation by a factor of 3, and a vertical translation by 5 units. What is the value of $g(-4)$?

(A) 266

(B) 170

(C) 28

(D) 20

The function g is given by $g(x) = x^3 - 3x^2 - 18x$, and the function h is given by $h(x) = x^2 - 2x - 35$. Let k be the function given by $k(x) = \frac{h(x)}{g(x)}$. What is the domain of k ?

(A) all real numbers x where $x \neq 0$

(B) all real numbers x where $x \neq -5$, $x \neq 7$

(C) all real numbers x where $x \neq -3$, $x \neq 0$, $x \neq 6$

(D) all real numbers x where $x \neq -5$, $x \neq -3$, $x \neq 0$, $x \neq 6$, $x \neq 7$